Challenges and Issues of Modern Science
2025, Vol. 4, No. 1, 83–88
ISSN: 3083-5704
URL: https://cims.fti.dp.ua/j/article/view/280
PURL: https://purl.org/cims/4.280

DOI: https://doi.org/10.15421/cims.4.280
UDC 519.71

Transfer Function of a Time-Varying Control System Considering Actuator Inertia

Volt Avdieiev

Purpose. Methodological support for building an algorithm for determining the transfer function (TF) of a link, which, considering the actuator dynamics and the disturbed motion of the mass center, is equivalent on a selected trajectory section to a time-varying control system (TCS) for the rocket movement in one plane. **Design / Method / Approach**. TCS is modeled using differential equations with changing coefficients. To define the type of TF, the Laplace transformation of the equations is performed, while its coefficients are determined by finding the equivalence criterion extreme of the output signals of the TCS and the link under the action of the test signal. **Findings.** The example of the TCS for the rocket movement in the yaw plane shows the possibility of an algorithm constructing for studying its dynamic characteristics by using the mathematical apparatus of linear stationary systems. **Theoretical Implications**. Finding the extreme of the equivalence criterion of the TCS and the link using the Levenberg-Marquardt method, with the coordinates of the extreme point being the arguments of the TF coefficients. **Practical Implications**. Using the TF of equivalent link, it is possible to obtain for the selected trajectory section a quantitative estimate of the stability margin, the duration of the transient process, the accuracy of disturbance compensation, and the transmission coefficient depending on the signal frequency input. The obtained results contribute to the methodological base expansion for linear time-varying systems research. **Originality / Value**. Analytical solution of the link differential equation for a test signal in the form of a sequence of rectangular and parabolic pulses using the Laplace transform. This will make it possible to obtain estimates of individual indicators of systems with time-varying parameters by using the mathematical apparatus of stationary systems. **Research Limitations / Future Research**. The algorithm is for the case of TCS of a rocket motion in one plane developed. The next stage of the study is to assess the algorithm complexity level as the order of the TCS mathematical model increases. **Article Type**. Methodological.

Keywords:

time-varying control system, transfer function, equivalence criterion

---

Мета. Методичне забезпечення побудови алгоритму визначення передатної функції (ПФ) ланки, яка з урахуванням динаміки виконавчого пристрою та збуреного руху центру мас є еквівалентною на обраній ділянці траєкторії нестаціонарній системі керування (НСК) рухом ракети в одній площині. **Дизайн / Метод / Підхід.** Модель НСК це диференційні рівняння зі змінними коефіцієнтами. Для визначення типу ПФ проводиться перетворення рівнянь за Лапласом, а її коефіцієнти визначають шляхом знаходження екстремуму критерію еквівалентності вихідних сигналів НСК і ланки під дією тестового сигналу. **Результати**. На прикладі НСК рухом ракети у площині рискання показана можливість побудови алгоритму дослідження її динамічних характеристик шляхом використання математичного апарату лінійних стаціонарних систем. **Теоретичне значення**. Використання методу Левенберга-Марквадта для знаходження екстремуму критерію еквівалентності НСК і ланки, координати екстремальної точки якого є аргументами коефіцієнтів ПФ. **Практичне значення**. Спираючись на ПФ еквівалентної ланки, можна отримати для вибраної дільниці траєкторії кількісну оцінку запасу стійкості, тривалості перехідного процесу, показників точності компенсації збурень і коефіцієнта передачі залежно від частоти вхідного сигналу. Отримані результати сприяють розширенню методичної бази дослідження лінійних нестаціонарних систем. **Оригінальність / Цінність**. Аналітичне рішення диференційного рівняння ланки при тестовому сигналі у вигляді послідовності імпульсів прямокутної і параболічної форми з використанням перетворення Лапласа. Це дасть можливість отримати оцінки окремих показників систем із змінними у часі параметрами шляхом використання математичного апарату стаціонарних систем. **Обмеження дослідження / Майбутні дослідження**. Алгоритм розроблено для НСК ракети в одній площині. Наступний етап дослідження це оцінка рівня складності алгоритму розрахунків при збільшенні порядку математичної моделі НСК. **Тип статті**. Методична.

Ключові слова:

нестаціонарна система керування, передатна функція, критерій еквівалентності

Contributor Details:

Volt Avdieiev, Dr. Sc., Prof., Oles Honchar Dnipro National University: Dnipro, UA, voltavde@i.ua

Received: 2025-04-24 Revised: 2025-05-07 Accepted: 2025-05-09

Time-varying control system (TCS) are a wide range of processes, ranging from rocket and space technology, production technology, turbofan engines, etc.; their analysis and synthesis is a complex mathematical problem, the solution of which has so far been obtained only for individual cases (Stenin at al., 2023). Most works on TCS in various versions of the problem statement consider the issue of synthesizing the optimal control law and ensuring stability. A sample data management algorithm has been developed for the case of linear TCSs, an asymptotic stability criterion has been substantiated (Zhang et al., 2019), and a stability criterion for the class of systems with piecewise constant parameters has been derived (Briat, 2015).

The traditional mathematical apparatus for analyzing linear stationary systems, for example, the Laplace transform, TF, characteristic polynomial, and frequency response, is used in the study of TCS, where the change in parameters depending on time has known limitations. This makes it possible to obtain approximate estimates of individual dynamic characteristics, in particular, the stability margin, the type and duration of the transient process of disturbance compensation. It is shown that uniform complete controllability of continuous TCS is equivalent to the possibility of arbitrary placement of the characteristic polynomial roots. The main components of the proof are the reduction of the system to an upper triangle and the use of the concept of uniform complete stabilization (Babiarz et al., 2021).

Using the example of a TCS with rocket rotational motion in one plane without taking into account the actuator inertia and the disturbed motion of the mass center, the possibility of using the Laplace transform to determine the amplitude stability margin and the phase stability margin is shown. The variable components of the model coefficients depending on time are presented in the form of a polynomial (Avdieiev & Alexandrov, 2023) and in the form of a sum of exponential functions (Avdieiev & Alexandrov, 2024). For the given data example, the error in determining these indicators is within 20–30%. An algorithm for calculating the TF coefficients of a second-order link is proposed, which is equivalent to the TCS in terms of dynamic characteristics on the selected section of the trajectory (Avdieiev, 2024). Algorithms for the TCS synthesis, some of the model parameters of which are in the uncertainty zone have been developed using the mathematical apparatus of linear matrix inequalities, (Nguyen & Banjerdpongchai, 2011), observation devices (Akremi et al., 2023), and sensor signals of the state vector individual coordinates (Avdieiev, 2021).

The example of a spacecraft orientation system with a magnetic drive shows the effectiveness of using Lyapunov differential equations in terms of ensuring stability indicators and finding a compromise between the adjustment time and the power requirements of the control system (Zhou, 2021). Based on the mathematical apparatus of Lyapunov functions, various approaches to ensuring the stability indicators of TCS have been developed. In particular, the inverse Lyapunov theorem for asymptotic stability has been proven (Kawano, 2020), an eigenvalue criterion has been proposed, and a condition for linear matrix inequalities has been obtained, which, compared with existing results, expands the range of TCS characteristics for which the obtained indicators retain their values (Chen & Yang, 2016).A systematic method for constructing Lyapunov functions for scalar linear systems is proposed, and a stability criterion for systems with piecewise constant parameters is proved (Zhou et al., 2020). It is proved that the TCS asymptotic stability occurs under the condition of negative real parts of the matrix eigenvalues and a certain limit on the rate of the parameters change, as well as under complete controllability (Guo & Rugh, 1995). It is shown that the complete TCS controllability implies the existence of feedback, and its connection with the Lyapunov exponent in stability theory is established (Anderson et al., 2013).

In addition to the requirement of a given stability margin, the TCS is required to ensure accuracy with the limited actuator power. The synthesis of optimal control laws for time-varying objects in the general case is a complex problem that cannot be solved analytically, which is associated with the solving complexity of the vector-matrix Riccati equation. An approach to solving the problem of the control law synthesis for one class of linear TCSs is proposed, which is based on the Pontryagin maximum principle. To establish the connection between the auxiliary vector and the state vector, the fundamental matrix of the system of simplified equations is used, which is determined by using the mathematical apparatus of Walsh functions. Since the mathematical model parameters are piecewise constant functions, it becomes possible to significantly simplify their practical implementation compared to matrices obtained based on the Riccati equation (Stenin et al., 2019).

Predictive control with model is a proven method to achieve optimal performance for linear system with constant parameters, while for time-varying one its use requires significant complications. An approximate optimal solution to the problem of predictive control of a non-stationary system for the Q-LPV class is proposed (Mate et al., 2023).

As is known, despite its high performance, predictive control requires significant computational resources, which complicates its implementation. The latest approach to this problem solving is to use a strategy that provides a solution to the control problem with limited computational capabilities. An example of its implementation is given for discrete TCS (Amiri & Hosseinzadeh, 2025).

The possibility of using the developed mathematical apparatus of stationary systems for studying TCS by rocket motion is provided by the method of frozen coefficients, known in the last century, according to which the coefficients of the TCS model in a small interval of a selected trajectory point are taken as constant. The disadvantage of this method is the dependence of the obtained estimates on the distance of the interval point to its middle.

The algorithm for determining a second-order stationary link, which in terms of dynamic characteristics is equivalent to the TCS of the rocket motion on a selected trajectory section, was proposed in work (Avdieiev, 2025), where the average quantitative assessment of equivalence for the section is found by iteration. This work does not consider the actuator inertia and the disturbed motion of the mass center in the direction perpendicular to the trajectory plane, which reduces the estimates reliability of the TCS dynamic characteristics, in particular, the size of its stability region in the space of the control law coefficients.

Analysis of available sources shows that most of them are devoted to obtaining theoretical results, while the development of methodological support of applied value for the design of aircraft motion control systems, in particular missiles and spacecraft, is not given due attention. This work sets the task of developing a methodological support for constructing an algorithm for determining the transfer function of a link, which, taking into account the actuators inertia and the disturbed motion of the mass center, is equivalent to a time-varying control system of missile motion in one plane on a selected trajectory section. This allows us to use the mathematical apparatus of stationary systems to estimate the stability margin, static error of disturbance compensation, and other indicators.

Problem statement

The TCS equation for rocket motion in one plane, for example, yaw, considering the actuator inertia and the disturbed motion of the mass center (Avdieiev, 2021):

$$\dot{\mathrm{x}} = \mathrm{a}(t) \cdot \mathrm{x} + \mathrm{f}(t), \tag{1}$$

where

$$\mathrm{a}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a_{\psi\psi}(t) & 0 & 0 & a_{\psi\delta}(t) & 0 \\ a_{z\psi}(t) & 0 & 0 & a_{z\delta}(t) & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \mu \cdot k_\psi & \mu \cdot k'_\psi & \mu \cdot k'_z & -\mu & -\mu \cdot \xi \cdot T \end{bmatrix},$$

$$\mathrm{f}(t) = \mathrm{c} \cdot f_z(t), \quad \mathrm{c} = [0 \ k_m \ 1 \ 0 \ 0]^T;$$

$$\mathrm{x} = \begin{bmatrix} \psi & \dot{\psi} & V_z & \delta & \dot{\delta} \end{bmatrix}^T,$$

where $a_{\psi\psi}(t)$, $a_{\psi\delta}(t)$, $a_{z\psi}(t)$, $a_{z\delta}(t)$ are parameters of the TCS model depending on time t; ξ, T are damping coefficient and time constant of the actuator; $f_z(t)$ is a perturbing acceleration of the rocket mass center; k_m is a coefficient that takes into account the distance between the mass center and the point of application of the resultant aerodynamic forces, as well as the ratio between the moment of inertia and the mass; k_ψ, k'_ψ, k'_z are control law coefficients; $\mu = 1/T^2$. The coordinates of the vector x are the following quantities: ψ, $\dot{\psi}$ are yaw angle and its derivative with respect to time; V_z is a projection of the velocity of the disturbed motion of the mass center onto the axis perpendicular to the trajectory plane; $\delta,\ \dot{\delta}$ are an equivalent angle of the actuator rudder rotation the and its derivative. If in a small neighborhood of the selected trajectory point the elements of the matrix a in equation (1) are considered constant, then the equation (1) can be transformed by Laplace, and five TF can be obtained:

$$w_i(s) = \frac{x_i(s)}{f_z(s)} = \frac{M_i(s)}{Q(s)} = \frac{\sum_{j=0}^{3} qm_{ij} \cdot s^j}{s^5 + \sum_{j=0}^{4} q_j \cdot s^j},\ i = 1\ldots5 \qquad (2)$$

The task is to develop a methodological support for constructing an algorithm for determining a link that, from the point of view of the selected criterion, is equivalent to the TCS of the rocket's motion in one plane on a certain trajectory section and has a TF of the form (2).

The problem solution

Based on (1), the coefficients of the characteristic polynomial $Q(s)$ in a small neighborhood of time t depending on the elements of the matrix $\mathbf{a}(t)$ are as follows:

$$q_0(t) = \mu \cdot k_z' \cdot [a_{\psi_\psi}(t) \cdot a_{z\delta}(t) - a_{\psi\delta}(t) \cdot a_{z\psi}(t)],$$

$$q_1(t) = -\mu \cdot [a_{\psi\psi}(t) + a_{\psi\delta}(t) \cdot k_\psi],$$

$$q_2(t) = -\mu \cdot [a_{z\delta}(t) \cdot k_z' + a_{\psi\delta}(t) \cdot k_\psi' + \varsigma \cdot T \cdot a_{\psi\psi}(t)],$$

$$q_3(t) = \mu - a_{\psi_\psi}(t),\ q_4 = \mu \cdot \varsigma \cdot T. \qquad (3)$$

The sequence of actions for determining the coefficients of the TF of the form (2) does not depend on the number of the vector **x** coordinate, so let's consider it using the example of the coordinate $x_1=\psi$. Based on model (1), the coefficients of the numerator of the TF $w_1(s)$ in a small neighborhood of time t are determined:

$$qm_0(t) = \mu \cdot k_z'[a_{\psi\delta}(t) - k_m \cdot a_{z\delta}(t)],$$

$$qm_1 = k_m \cdot \mu,\ qm_2 = k_m \cdot \mu \cdot \varsigma \cdot T,\ qm_3 = k_m \qquad (4)$$

A link with a TF of the form (2) is taken to be equivalent to a TCS on a certain trajectory section in terms of dynamic characteristics, when the criterion for the difference of the output signals of the TCS and the link in the searching process in the four-dimensional space of the quantities $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\delta}$, $a_{z\psi}$ will take a minimum value. The coordinates of the output signals vector are the yaw angle ψ and its four time derivatives. The linear differential equation of the link that follows from TF (2), for the coordinate ψ, is as follows:

$$\psi(t)^{(5)} + \sum_{i=0}^{4} q_i \cdot \psi(t)^{(i)} = \sum_{i=0}^{3} qm_i \cdot f_z(t)^{(i)}, \qquad (5)$$

where the superscripts define the time derivative of the corresponding order.

The input signal $f_z(t)$, necessary for the emergence of a transient process of disturbance compensation, depends, in particular, on the estimate of the transient process duration at the midpoint of the selected trajectory interval. The paper considers a variant of the disturbance $f_z(t)$ at the input of TCS and at the input of the link with the TF of the form (2) as a sequence of four pulses: figure 1 – test signal as rectangular pulses, figure 2 – test signal as pulses in the shape of parabola. As a equivalence criterion the TCS and the link, we take the average on the selected trajectory section for n moments of time the value of the modulus of the coordinates difference of the TCS output signal vector ψz and the link output signal vector ψa with the signal $f_z(t)$ at their inputs. The vector ψz at n points of the trajectory section is the result of the numerical solution of equation (5) considering the time dependence of the coefficients (3, 4) and remains constant in the process of finding the minimum criterion. The vector ψa at n points of the trajectory section is determined by analytically solving equation (5) depending on the values $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\delta}$, $a_{z\psi}$, which vary in the process of the criterion minimum finding. The presence of an analytical solution significantly reduces the duration of the iterative process of the minimum finding. Thus, the equivalence criterion of the TCS and the link can be written as

$$R(a_{\psi\psi}, a_{\psi\delta}, a_{z\psi}, a_{z\delta}) =$$

$$= \frac{1}{n} \sum_{i=1}^{n} \sum_{k=1}^{5} \left| \psi z_{ik} - \psi a_{ik}(a_{\psi\psi}, a_{\psi\delta}, a_{z\psi}, a_{z\delta}) \right| / xm_k, \qquad (6)$$

here xm is an array of numbers for transition to dimensionless coordinates.

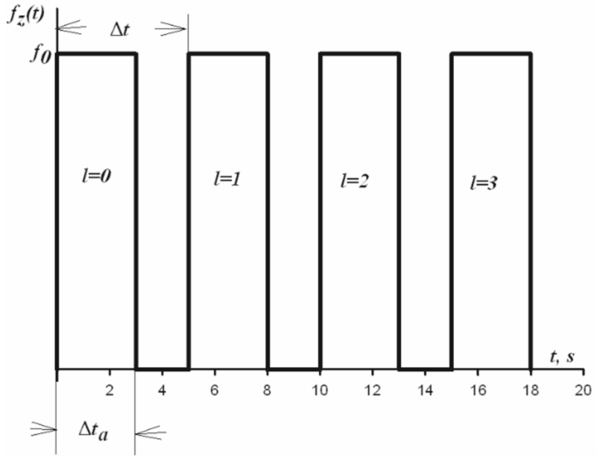

Figure 1 – Test signal as rectangular pulses (Source: author)

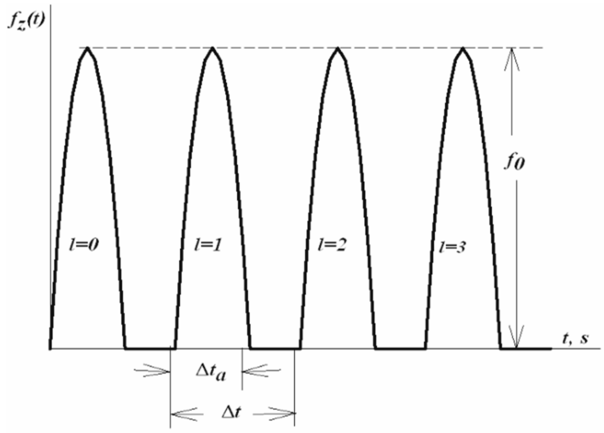

Figure 2 – Test signal as pulses in the shape of parabola (Source: author)

As a result of the criterion (6) minimum found in the four-dimensional space $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\psi}$, $a_{z\delta}$ the TF coefficients of the form (2) qm_0, q_0, q_1, q_2, q_3 are determined, which in the relations (3, 4) depend on time. The search for the minimum of criterion (6) is carried out by the Levenberg-Marquardt method, the efficiency of which in terms of time consumption depends on the choice of the algorithm for the analytical solution of the differential equation (5) for the test signal $f_z(t)$ (Fig. 1, 2). According to Fig. 1

$$f_z(t) = \begin{cases} f_0 & 0 \le t - \Delta t \cdot l \le \Delta t_a \\ 0 & \Delta t_a < t - \Delta t \cdot l < \Delta t \end{cases} \quad l = 0\ldots3, \qquad (7)$$

where l, Δt_a are pulse number and its duration; $4\Delta t$ is an interval of the trajectory section to which the equivalent link corresponds.

The Laplace transform of the differential equation (5) with zero initial values and a constant perturbation f_0 gives the image of the first coordinate of the link output signal vector

$$\psi(s) = \frac{qm_0 \cdot f_0}{s \cdot Q(s)}.$$

As is known from the operational calculus theory, the original of this image

$$\psi(t) = qm_0 \cdot f_0 \cdot \left(\frac{1}{q_0} + \sum_{k=0}^{4} \frac{e^{s_k \cdot t}}{s_k \cdot dQ_s(s_k)} \right),$$

$$(8)$$

$$dQ_s(s_k) = \frac{dQ}{ds}(s_k) = 5s_k^4 + \sum_{j=0}^{3} q_{j+1} \cdot (j+1) \cdot s_k^j$$

where s_k is the root of the polynomial $Q(s)$ with number k.

The next four coordinates of the output signal vector of the link

$$\psi^{(i)}(t) = qm_0 \cdot f_0 \cdot \left(\sum_{k=0}^{4} \frac{s_k^{i-1} \cdot e^{s_k \cdot t}}{dQ_s(s_k)} \right),\ i = 1\ldots4. \qquad (9)$$

The relation (8, 9) is an analytical solution of equations (5) for the input signal (7), but when the signal shape changes (Fig. 1, 2), the initial values components, which are the result of the solution for the previous shape, should be added to it.

Based on the operational calculus rules, the image component of the first coordinate of the output signal vector of the link taking in account five initial values $\psi_0^{(i)}$

$$\psi_{st}(s) = \frac{F(s)}{Q(s)}, \quad F(s) = \sum_{k=0}^{3} b_k \cdot s^k + s^4 \cdot \psi_0^{(0)}, \quad (10)$$

$$b_k = \psi_0^{(4-k)} + \sum_{i=0}^{3-k} q_{i+k+1} \cdot \psi_0^{(i)}.$$

Original of image (10) according to the operational arithmetic rules

$$\psi_{st}^{(0)}(t) = \sum_{k=0}^{4} \frac{F(s_k)}{dQ_s(s_k)} \cdot e^{s_k \cdot t}. \quad (11)$$

The following four components determined by the initial values of the coordinates of the vector of the link's output signal:

$$\psi_{st}^{(i)}(t) = \sum_{k=0}^{4} \frac{F(s_k) \cdot s_k^i}{dQ_s(s_k)} \cdot e^{s_k \cdot t}, \quad i = 1...4. \quad (12)$$

The obtained analytical solution (8-12) of the differential equation (5) considering the initial values, which are updated when the shape of the test disturbance signal (7) changes, is used to find the minimum of criterion (6) by the Levenberg-Marquardt method in the four-dimensional space of quantities $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\psi}$, $a_{z\delta}$.

For the case of a test signal $f_z(t)$ in the form of a sequence of pulses of parabolic form (Fig. 2), the analytical solution of the differential equation (5) can also be obtained using the operational calculus rules.

In the interval of one pulse, the test signal is a parabola

$$f_z(\tau) = a\tau^2 + b\tau + d, \ \tau = 0...\Delta t_a, \quad (13)$$

where $a = \frac{-4f_0}{\Delta t_a^2}, \ b = \frac{4f_0}{\Delta t_a}, \ d = 0.$

From TF (2) Laplace transform of the yaw angle

$$x_1(s) = \psi(s) = f_z(s) \cdot M_1(s)/Q(s), \quad (14)$$

$$M_1(s) = \sum_{k=0}^{3} qm_k \cdot s^k$$

Differential equation of the equivalent link according to (13, 14)

$$\psi^{(5)} + \sum_{k=0}^{4} q_k \cdot \psi^{(k)} = qm_0 \cdot (a\tau^2 + b\tau) +$$
$$+qm_1 \cdot (2a\tau + b) + 2qm_2 \cdot a =$$
$$= v_0 + v_1 \cdot \tau + v_2 \cdot \tau^2. \quad (15)$$

The variant of sequential actions for obtaining the solution of equation (15) is as follows.

The Laplace transformation of (15) gives

$$\psi(s) \cdot Q(s) = \frac{v_0 \cdot s^2 + v_1 \cdot s + 2v_2}{s^3} = \frac{P(s)}{s^3},$$

therefore, the second derivative of the solution

$$\ddot{\psi}(\tau) = \psi^{(2)}(\tau) = L^{-1}\left\{\frac{P(s)}{s \cdot Q(s)}\right\} =$$
$$= \frac{2v_2}{q_0} + \sum_{k=0}^{k=4} \frac{P(s_k)}{s_k \cdot dQ_s(s_k)} \cdot e^{s_k \cdot \tau},$$

where the symbol L^{-1} means the inverse Laplace transform, i.e. the transition from the image to the original.

Higher-order derivatives according to the operational calculus rules

$$\psi^{(l)}(\tau) = \sum_{k=0}^{4} \frac{P(s_k) \cdot s_k^{l-3}}{dQ_s(q, s_k)} \cdot e^{s_k \cdot \tau}; \ l = 3,4; \quad (16)$$

First and zero order derivatives

$$\dot{\psi}(\tau) = \int_0^\tau \ddot{\psi}(\tau_1) \cdot d\tau_1 =$$
$$= \frac{2v_2}{q_0} \cdot \tau + \sum_{k=0}^{4} \frac{B_k \cdot e^{s_k \cdot \tau_1}}{s_k} \Big|_0^\tau =$$
$$= \frac{2v_2}{q_0} \cdot \tau + \sum_{k=0}^{4} \frac{B_k \cdot (e^{s_k \cdot \tau} - 1)}{s_k}, \quad (17)$$

$$B_k = \frac{P(s_k)}{s_k \cdot dQ_s(q, s_k)},$$

$$\psi(\tau) = \psi^{(0)}(\tau) = \int_0^\tau \dot{\psi}(\tau_1) \cdot d\tau_1 =.$$
$$= \frac{v_2 \cdot \tau^2}{q_0} + \sum_{k=0}^{4} \left\{\frac{B_k}{s_k^2} \cdot (e^{s_k \cdot \tau} - 1) - \frac{B_k}{s_k} \cdot \tau\right\}. \quad (18)$$

The obtained solutions (16-18) of equation (15) for the case of a parabolic test pulse (Fig. 2) can be used to determine the output signal of the equivalent circuit for a sequence of test pulses in the form of a parabola, taking into account the initial conditions when changing the waveform similarly to a sequence of rectangular pulses.

Let us consider the definition of the link, which on the selected trajectory interval is equivalent to the TCS of the rocket motion in the yaw plane, using the data example dependences on the time of the model (1) coefficients (Source: author): $a_{\psi\psi}(t)$ – figure 3, $a_{\psi\delta}(t)$ – figure 4, $a_{z\psi}(t)$ – figure 5, $a_{z\delta}(t)$ – figure 6. As is known, the instability in time of the model (1) coefficients is caused by a change in the rocket mass-inertial characteristics, speed and flight altitude.

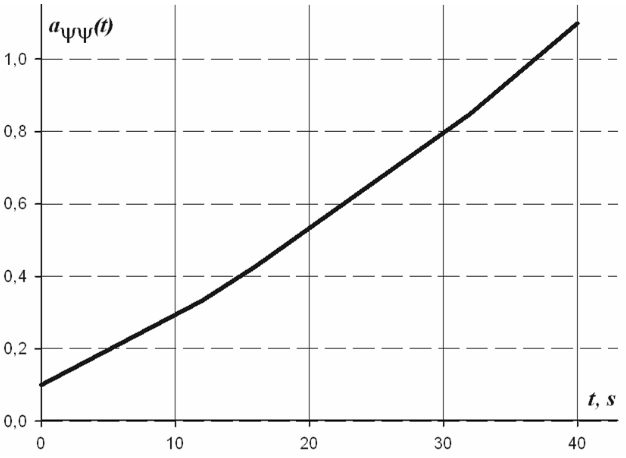

Figure 3 (Source: author)

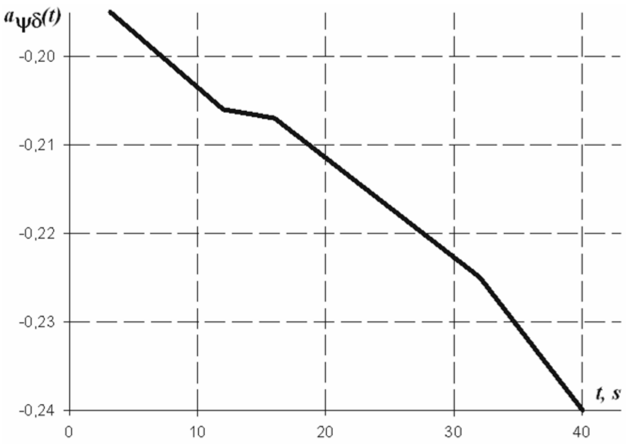

Figure 4 (Source: author)

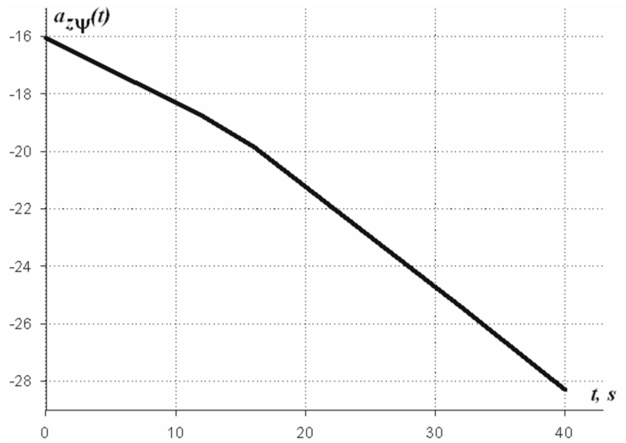

Figure 5 (Source: author)

To substantiate the possibility of the algorithm constructing for determining a link that is equivalent from the point of view of the selected criterion to the TCS of the rocket's motion on a certain trajectory section, an experiment was conducted using the data in Fig. 3-6 and Table 1.

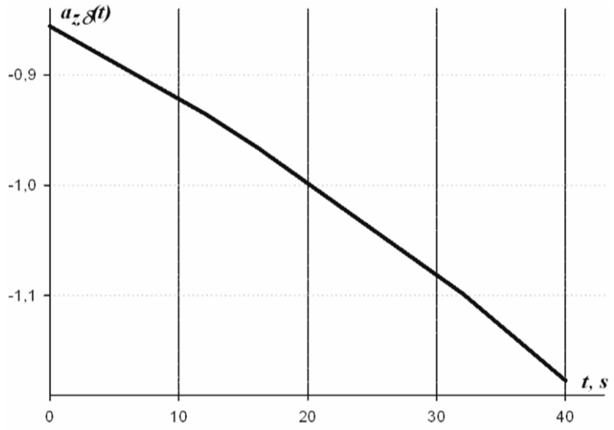

Figure 6 (Source: author)

Table 1 – actuators parameters and control law coefficients (Source: author)

T	ξ	k_ψ	k'_ψ	k'_z
s	-		s	s/м
0.1	1.2	23.74	19.89	-0.858

For the test signal $f_z(t)$ in the form of four rectangular pulses, as a result of finding the criterion (6) minimum by the Levenberg-Marquardt method in the four-dimensional space of quantities $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\psi}$, $a_{z\delta}$ using relations (3, 4), the TF of the form (2) are determined:

$$w_{\psi f}(s) = \frac{\psi(s)}{f_z(s)} = \frac{qm_0}{s^5+\sum_{k=0}^{4} q_k \cdot s^k} =$$
$$= \frac{11.71}{s^5+12s^4=99.79s^3+313.9s^2+448.4s+317.5}. \quad (19)$$

The link with TF (19) according to criterion (6) is equivalent to TCS (1) in the relative time interval 0…20 s.

For the case of a test signal $f_z(t)$ in the form of pulses of a parabolic shape (Fig. 2), the TF of the link, which in the relative time interval t = 0…20 s is equivalent to the TCS (1), is determined according to the searching results for the minimum of criterion (6) and relations (3, 4):

$$w_{\psi f}(s) = \frac{\psi(s)}{f_z(s)} = \frac{qm_0}{s^5+\sum_{k=0}^{4} q_k \cdot s^k} =$$
$$= \frac{0.072s^3+0.86s^2+7.2s+11.85}{s^5+12s^4=99.97s^3+317.4s^2+453.1s+328.0}. \quad (20)$$

Calculations show that the criterion R (6) may have local extremes, the coordinates of which depend on their initial values and two-sided restrictions of the arguments $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\psi}$, $a_{z\delta}$, while the minimum value of R for the given data example in the local extremes is the same.

As can be seen from the comparison of TF (19) and (20), the difference between the denominator coefficients, which is a consequence of different test signals, for this example data does not exceed 3.5%, while the difference in the estimates of the stability margin on the roots plane of the characteristic polynomial is about 2%.

The following values are given at the algorithm input for determining the TF of the link, which is equivalent to the time-varying missile control system in one plane on the selected trajectory section, taking into account the actuator inertia and the disturbed mass center motion:

- constant coefficients ξ, T of the model (1), which quantitatively characterize the actuator speed;
- tables of time dependences of the coefficients $a_{\psi\psi}(t)$, $a_{\psi\delta}(t)$, $a_{z\psi}(t)$, $a_{z\delta}(t)$;
- the beginning and end moments of the selected trajectory section;
- the control law coefficients k_z', k_ψ, k_ψ', calculated for midpoint of the selected trajectory section;
- test signals for excitation of the transient process.

To the algorithm output are placed the TF coefficients of the kind (2) for the coordinate x_1=ψ.

The main steps are as follows:

- approximation of tabulated coefficients $a_{\psi\psi}(t)$, $a_{\psi\delta}(t)$, $a_{z\psi}(t)$, $a_{z\delta}(t)$ by polynomials;
- numerical solution of equation (5), which follows from model (1), using approximation polynomials and the selected test signal to excite the transient process;
- selection of the procedure for analytical solution of equation (5) or (15) depending on the test signal and possible values of the quantities $a_{\psi\psi}$, $a_{\psi\delta}$, $a_{z\psi}$, $a_{z\delta}$, which can be equivalent to the variable coefficients $a_{\psi\psi}(t)$, $a_{\psi\delta}(t)$, $a_{z\psi}(t)$, $a_{z\delta}(t)$ in equation (1);
- finding the minimum of the criterion R (6) by the Levenberg-Marquardt method, by using the appropriate procedure, for example, Minimize in the Mathcad software environment;
- calculation of the TF coefficients according to the relations (3, 4).

Based on the TF (19, 20) by methods of the theory of linear stationary systems, it is possible to obtain estimates of such dynamic characteristics of TCS as accuracy indicators, frequency characteristics, type of transient process and its duration, as well as to determine the influence of the actuator inertia on these characteristics. The presence of these indicators can be used to make technical decisions in the process of TCS's designing.

Conclusions

A methodological support for an algorithm constructing for determining the transfer function of a link, which on a selected trajectory section is equivalent to a linear time-varying control system for the movement of a rocket in one plane, taking into account the actuator inertia and the disturbed motion of the mass center, has been developed.

In particular, the following are proposed:

- test signal variants for the transient process excitation in order to obtain a sufficient amount of data for assessing the dynamic characteristics of the time-varying system;
- formulas for the analytical solution of the link differential equation using the mathematical apparatus of the Laplace transform for the test signal in the form of a sequence of rectangular and parabolic pulses, considering the initial conditions, which are updated when the waveform changes;
- a criterion for quantitatively assessing the difference between the output signals of a time-varying system and an equivalent link, the coordinates of the extreme point of which, found by the Levenberg-Marquardt method, are the arguments of the transfer function coefficients.

The novelty of the work lies in taking into account the actuator inertia and the disturbed motion of the mass center when developing a methodological support for an algorithm constructing for determining a stationary link, which, in terms of dynamic characteristics, is equivalent to the TCS movement of a rocket on a certain trajectory section.

Practical significance lies in supplementing the methodological base for designing time-varying systems by using the mathematical apparatus of stationary systems in terms of assessing dynamic characteristics, namely the stability margin on the plane of the characteristic polynomial roots, the accuracy of disturbance compensation, the type of transient process and its duration, as well as the amplitude-frequency and phase-frequency characteristics.

References

Akremi, R., Lamouchi, R., Amairi, M., Dinh, T. N., & Raïssi, T. (2023). Functional interval observer design for multivariable linear parameter-varying systems. *European Journal of Control, 71*, 100794. https://doi.org/10.1016/j.ejcon.2023.100794

Amiri, M., & Hosseinzadeh, M. (2025). Practical considerations for implementing robust-to-early termination model predictive control. *Systems & Control Letters, 196*, 106018. https://doi.org/10.1016/j.sysconle.2024.106018

Anderson, B. D. O., Ilchmann, A., & Wirth, F. R. (2013). Stabilizability of linear time-varying systems. *Systems & Control Letters, 62*(9), 747–755. https://doi.org/10.1016/j.sysconle.2013.05.003

Avdieiev, V. (2024). Transfer functions of a time-varying control system. *Challenges and Issues of Modern Science, 2*, 265-274. https://cims.fti.dp.ua/j/article/view/187

Avdieiev, V. (2025). Evaluation of dynamic characteristics of a linear time-varying system. *Challenges and Issues of Modern Science, 4*(1). https://cims.fti.dp.ua/j/article/view/250

Avdieiev, V. V. (2021). Determination of model parameters of rocket stabilization system in flight. *International Scientific Technical Journal "Problems of Control and Informatics", 66*(6), 78–92. https://doi.org/10.34229/1028-0979-2021-6-8

Avdieiev, V. V., & Aleksandrov, A. E. (2023). Missile movement control system stability reserve. *System Design and Analysis of Aerospace Technique Characteristics, 32*(1), 3–14. https://doi.org/10.15421/472301

Avdieiev, V. V., & Alexandrov, A. E. (2024). Margin of stability of the time-varying control system for rotational motion of the rocket. *Radio Electronics, Computer Science, Control, 3*, 185–195. https://doi.org/10.15588/1607-3274-2024-3-16

Babiarz, A., Cuong, L. V., Czornik, A., & Doan, T. S. (2021). Necessary and sufficient conditions for assignability of dichotomy spectra of continuous time-varying linear systems. *Automatica, 125*, 109466. https://doi.org/10.1016/j.automatica.2020.109466

Briat, C. (2015). Stability analysis and control of a class of LPV systems with piecewise constant parameters. *Systems & Control Letters, 82*, 10–17. https://doi.org/10.1016/j.sysconle.2015.05.002

Chen, G., & Yang, Y. (2016). New stability conditions for a class of linear time-varying systems. *Automatica, 71*, 342–347. https://doi.org/10.1016/j.automatica.2016.05.005

Guo, D., & Rugh, W. J. (1995). A stability result for linear parameter-varying systems. *Systems & Control Letters, 24*(1), 1–5. https://doi.org/10.1016/0167-6911(94)00013-I

Kawano, Y. (2020). Converse stability theorems for positive linear time-varying systems. *Automatica, 122*, 109193. https://doi.org/10.1016/j.automatica.2020.109193

Mate, S., Jaju, P., Bhartiya, S., & Nataraj, P. S. V. (2023). Semi-Explicit Model Predictive Control of Quasi Linear Parameter Varying Systems. *European Journal of Control, 69*, 100750. https://doi.org/10.1016/j.ejcon.2022.100750

Nguyen, D. H., & Banjerdpongchai, D. (2011). A convex optimization approach to robust iterative learning control for linear systems with time-varying parametric uncertainties. *Automatica, 47*(9), 2039–2043. https://doi.org/10.1016/j.automatica.2011.05.022

Stenin, A. A., Timoshin, Y. A., & Drozdovich, I. G. (2019). Walsh Functions in Linear-Quadratic Optimization Problems of Linear Nonstationary Systems. *Journal of Automation and Information Sciences, 51*(8), 43–57. https://doi.org/10.1615/jautomatinfscien.v51.i8.40

Stenin, A. A., Drozdovych, I. G., & Soldatova, M. O. (2023). Application of spline functions and walsh functions in problems of parametric identification of linear nonstationary systems. *Radio Electronics, Computer Science, Control, 2*, 166–175. https://doi.org/10.15588/1607-3274-2023-2-17

Zhang, W., Han, Q.-L., Tang, Y., & Liu, Y. (2019). Sampled-data control for a class of linear time-varying systems. *Automatica, 103*, 126–134. https://doi.org/10.1016/j.automatica.2019.01.027

Zhou, B. (2021). Lyapunov differential equations and inequalities for stability and stabilization of linear time-varying systems. *Automatica, 131*, 109785. https://doi.org/10.1016/j.automatica.2021.109785

Zhou, B., Tian, Y., & Lam, J. (2020). On construction of Lyapunov functions for scalar linear time-varying systems. Systems & Control Letters, 135, 104591. https://doi.org/10.1016/j.sysconle.2019.104591