Challenges and Issues of Modern Science
2025, Vol. 4, No. 1, 43-47
ISSN: 3083-5704
URL: https://cims.fti.dp.ua/j/article/view/299
PURL: https://purl.org/cims/4.299

DOI: https://doi.org/10.15421/cims.4.299

UDC 629.7.036

Modeling fin efficiency considering transverse temperature gradients in rocket engine cooling channels

Volodymyr Sliusariev, Valerii Bucharskyi

Purpose. This study aims to improve the accuracy of methods for determining fin efficiency. Its goal is to derive calculation relationships for finning coefficients that account for transverse temperature non-uniformity within the fin cross-section. **Design / Method / Approach.** The article presents the reduction of the heat conduction equation for a fin to a dimensionless form, based on dimensional analysis of the variables involved. Further development relies on analyzing the results of numerical simulations and their subsequent generalization. To this end, the gradient descent method is applied, minimizing the quadratic error function. **Findings.** A criterial dependence has been formulated to complement the derived heat conduction equation. Test calculations and comparisons with numerical simulations in Ansys Fluent confirm an improvement in calculation accuracy when using the proposed equation. **Theoretical Implications.** This paper addresses factors previously neglected in the analysis of heat transfer in fins. The results of the study thus complement existing approaches to determining finning coefficients. **Practical Implications.** The derived criterial relationship will enhance the accuracy of heat transfer calculations in the chambers and gas generators of liquid rocket engines. **Originality / Value.** The paper introduces an original criterial relationship that accounts for temperature non-uniformity across the fin cross-section. Incorporating this factor improves calculation accuracy, highlighting the practical value of the developed equation. **Research Limitations / Future Research.** This study focuses on rectangular fins; therefore, the proposed model is not applicable to fins with variable thickness in the ducts of liquid propellant rocket engines (LPREs). Developing a fin model without these limitations will be the objective of future research on this topic. **Article Type.** Applied Research.

Keywords:

mathematical model of heat transfer, transverse temperature non-uniformity, liquid propellant rocket engine, cooling channels of the engine chamber

---

Мета. Дана робота спрямована на підвищення точності методів визначення ефективності оребрення. Метою роботи є отримання розрахункових співвідношень для коефіцієнтів оребрення, що враховують температурну нерівномірність у поперечному перерізі ребра. **Дизайн / Метод / Підхід.** У статті представлені результати приведення рівняння теплопровідності в ребрі до безрозмірного виду на основі аналізу розмірностей величин, що входять до рівняння. Наступна частина роботи робота ґрунтується на аналізі результатів чисельного моделювання та їх подальшому узагальненні. Для цього використовується метод градієнтного спуску з урахуванням мінімізації квадратичної функції помилки. **Результати.** Розроблено критеріальну залежність, яка доповнює отримане рівняння теплопровідності у ребрі. Також проведено тестові розрахунки та порівняння з чисельним моделюванням у Ansys Fluent, що підтверджують підвищення точності розрахунку при використанні отриманого рівняння. **Теоретичне значення.** У роботі розглянуті фактори, що впливають на теплопередачу в ребрах, якими раніше нехтувалося. Таким чином, результати, отримані в дослідженні, доповнюють існуючі підходи до визначення коефіцієнтів оребрення. **Практичне значення.** Отримана критеріальна залежність дозволить підвищити точність розрахунків телепередачі у камерах та газогенераторах рідинних ракетних двигунів. **Оригінальність / Цінність.** Робота містить оригінальне критеріальне співвідношення, що дозволяє враховувати температурну нерівномірність у поперечному перерізі ребра. Урахування цього фактору дозволяє підвищити точність розрахунків, що зумовлює цінність отриманого рівняння під час проведення практичних розрахунків. **Обмеження дослідження / Майбутні дослідження.** У цьому дослідженні розглядаються прямокутні ребра. Відповідно, отримана модель не може бути використана для визначення коефіцієнта оребрення в трактах рідинних ракетних двигунів (РРД) з ребрами змінної товщини. Розробка моделі ребра, яка була б позбавлена даних обмежень, стане метою подальших досліджень з даної тематики. **Тип статті.** Прикладне дослідження.

Ключові слова:

математична модель теплопередачі, поперечна температурна нерівномірність, рідинний ракетний двигун, тракт охолодження камери двигуна

Contributor Details:

Volodymyr Sliusariev, PhD student, Oles Honchar Dnipro National University: Dnipro, UA, sliusariev@ftf.dnu.edu.ua
Valerii Bucharskyi, PhD, Assoc. Prof., Oles Honchar Dnipro National University: Dnipro, UA, bucharskyi@ftf.dnu.edu.ua

Received: 2025-05-26 *Revised: 2025-06-14* *Accepted: 2025-06-21*

Accurate calculation of the influence of fin parameters on heat transfer in the cooling tract is a critical stage in the design of liquid-propellant rocket engine chambers and has been the subject of extensive research in recent studies (Leonardi et al., 2019; Vekilov et al., 2021). This is primarily due to the fact that the fin efficiency coefficient can assume values either greater or less than one. Consequently, in some cases, the selected fin geometry may actually degrade heat transfer performance. To prevent such outcomes, reliable and precise methods for calculating all heat transfer parameters are essential.

However, many widely used approaches were developed during the last century. At that time, the ability to conduct a large number of computational experiments was limited, and several methods were deliberately simplified to allow for manual calculations without the use of computers. As a result, these established methods possess significant limitations regarding the ranges of parameters within which their application remains valid.

An example of such a technique is the traditional approach to fin calculations (Forsberg, 2021). In deriving the heat conduction equation for a fin, simplifications are introduced by treating the fin as a rod whose cross-sectional dimensions (a^2) are small compared to its length (L) (see Fig. 1), thereby reducing the problem to a one-dimensional heat conduction formulation.

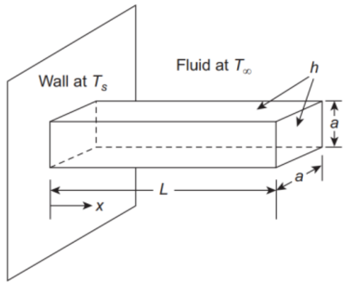

Figure 1 – Considered configurations (Source: Forsberg, 2021)

As a result, the equation incorporates an average temperature over the fin's cross-section. However, the intensity of heat transfer depends on the surface temperature of the fin, which, in the general case, differs from the cross-sectional average. Moreover, in real engines, the geometry of fins does not always align with these simplifying assumptions. Therefore, applying this equation can lead to significant errors in certain cases.

An alternative approach involves developing new mathematical models based on the generalization of numerical experiment results. Such models, on the one hand, overcome the limitations of earlier methods, and on the other hand, enable the rapid determination of cooling system parameters – an important advantage in the context of growing competition in the rocket and space industry. This is supported by studies (Sliusariev & Bucharskyi, 2024; Bucharskyi et al., 2018), which demonstrate that new mathematical models can achieve improved accuracy while reducing the computational effort required.

Naturally, the resulting mathematical models are less convenient for manual use, but they are better suited for algorithmic implementation and programming. Therefore, a key consideration in this context is the effective use of computer-integrated technologies in rocket engine design, as illustrated in (Sukachevskyi & Shevtsov, 2024). At the initial stage of model development, it is important to determine the target platform for implementation—whether it will be a computer algebra system such as Mathcad (PTC Inc, 2025), or specialized libraries for general-purpose programming languages, such as SciPy for Python (Virtanen, P et al., 2020). This helps to identify the available capabilities and limitations from the outset. In their work, the authors focused on the functionality provided by the Wolfram Mathematica system (Wolfram Research, Inc., 2025).

It is also important to note that such a model is applicable not only to LPRE chambers, but also to other components, such as gas generators and heat exchangers in tank pressurization systems (Mitikov & Sedchenko, 2023). The results obtained may also prove valuable in the development of new heat supply systems, whose significance is highlighted in recent studies (Bilohurov et al., 2024; Tokarskyi & Habrinets, 2024).

Literature review and problem statement

To confirm the relevance and novelty of the problem, a review of recent research in this area was conducted. The literature analysis revealed that many contemporary studies are still based on classical finning theory. Examples include studies (Sichler et al., 2018; Kose & Celik, 2023), which present the development of one-dimensional models for cooling rocket engine chambers using methane – a particularly timely topic given the growing interest in this fuel, driven in part by the success of SpaceX's Raptor engine (Williams, 2023).

Another widely adopted approach involves the use of computer-aided engineering (CAE) systems. These tools enable the simulation of both hydraulic processes within cooling channels and heat conduction through chamber walls and fins. For instance, study (Jeong et al., 2023) demonstrates the feasibility of modeling heat transfer using open-source software, which offers a cost-effective alternative for small companies and startups unable to invest in commercial solutions. In other studies (Xu et al., 2023; Kim et al., 2014), heat conduction in the chamber wall and fins was modeled as part of a conjugate heat transfer analysis of the engine chamber. However, it is important to note that numerical modeling requires substantial computational resources and remains time-consuming, even with modern tools.

An innovative approach was proposed in (Fagherazzi et al., 2023), where a novel multi-zone method was applied to calculate heat conduction within the fins. This technique improved the accuracy of the results, though it came at the cost of increased computational complexity. The authors validated their model by comparing its predictions with data from hot-fire tests of an engine chamber.

Recent studies also explore the optimization of cooling duct geometry. In (Lv et al., 2023), temperature fields in the chamber walls – obtained through three-dimensional numerical simulations – were analyzed to guide the optimization of channel parameters. Another study (Atefi & Naraghi, 2019) formulated an optimization problem based on classical methods for evaluating heat transfer intensity, finning efficiency, and pressure losses. Using an iterative approach, the authors determined duct geometries that minimized either temperature variations along the duct or the maximum wall temperature.

However, the literature review also revealed a lack of sufficiently simple analytical relationships that account for transverse temperature non-uniformity. Such models would be particularly useful during early-stage design and for conducting parametric studies. In light of these considerations, the development of a mathematical model that accounts for transverse temperature non-uniformity in the fins of LRE cooling channels remains a relevant task.

Objective and Tasks

The objective of this study is to develop an approach for calculating the finning coefficient that accounts for the reduction in heat transfer intensity from the fin surface due to transverse temperature non-uniformity within its cross-section.

To achieve this objective, the following tasks are addressed:

– reformulate the heat conduction equation for a straight fin into a dimensionless form by deriving the characteristic dimensionless criteria;

– perform numerical simulations of heat conduction in fins and assess the error in the resulting equation associated with temperature non-uniformity;

– derive a criterial relationship for the heat conduction equation in the fin that incorporates the effect of temperature non-uniformity.

Materials and Methods

In this study, the theory of dimensions and similarity was employed to reformulate the heat conduction equation in a dimensionless form.

The influence of transverse temperature non-uniformity in the fin cross-section was evaluated using the results of numerical simulations. Thermal conductivity modeling was carried out in the CAE system Ansys Fluent 2020 R1. In these simulations, the fin thickness (δ), material thermal conductivity (λ), and the heat transfer coefficient (α) were varied. The considered cases, along with the problem setup, are presented in Figure 2.

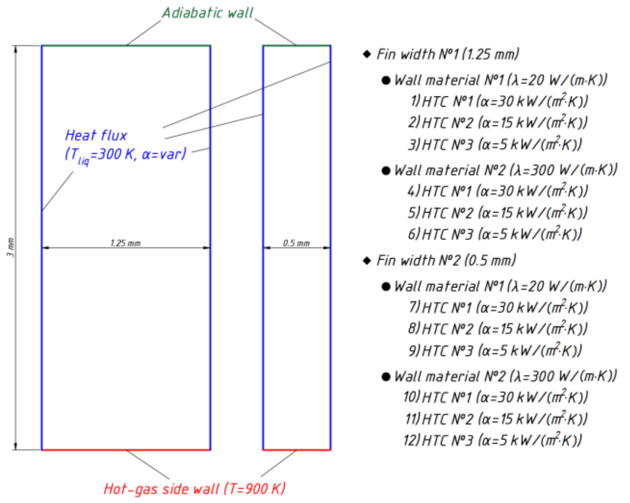

Figure 2 – Considered configurations (Source: Authors)

To derive the criterion-based correlation from the numerical simulation results, regression analysis (Mohr et al., 2022) was performed using logarithmic scaling. This approach is applicable to a relationship of the form:

$$u = C\, z^{m}. \tag{1}$$

After that, the logarithms of both sides of the equation are taken:

$$\ln(u) = \ln(C) + m\,\ln(z),$$

resulting in a linear regression model:

$$\hat{y} = a + m\,x, \tag{2}$$

where $\hat{y} = \ln(\mathrm{u})$, $a = \ln(C)$, $x = \ln(z)$.

Next, the coefficients *a* and *m* were determined using the gradient descent method by minimizing the quadratic error function. To achieve this, we define the following function:

$$J(a,m) = \frac{1}{2\,N}\sum\left(\hat{y}^{(i)} - y^{(i)}\right)^2,$$

where *N* – number of data points used for the regression, *y* – represents the known function values for these data points, $\hat{y}$ – value predicted by equation (2) using the current coefficients *a* and *m*. At each iteration, the coefficients a and m are updated according to the following expressions:

$$a \leftarrow a - \eta\,\frac{\partial J}{\partial a}; \tag{3}$$

$$m \leftarrow m - \eta\,\frac{\partial J}{\partial m}. \tag{4}$$

Here, η – learning rate, chosen as the maximum value that ensures numerical stability. The partial derivatives in equation (3, 4) are defined as follows:

$$\frac{\partial J}{\partial a} = \frac{1}{N}\sum\left(a + m\,x^{(i)} - y^{(i)}\right)\,x^{(i)};$$

$$\frac{\partial J}{\partial m} = \frac{1}{N}\sum\left(a + m\,x^{(i)} - y^{(i)}\right).$$

The calculation was performed iteratively until the coefficients *a* and *m* converged.

Results

This study considered ribs of constant thickness. To formulate the corresponding equation, the relationship proposed by the authors in (Sliusariev & Bucharskyi, 2024) was taken as a starting point:

$$\frac{d}{dx}\left(\lambda\,\delta\,\frac{dT}{dx}\right) = \alpha\left(T - T_{liq}\right)\sqrt{1 + \left(\frac{d\delta}{dx}\right)^2},$$

a transformation was then applied to express the temperature in terms of the excess temperature relative to the liquid temperature:

$$\theta = T - T_{liq}.$$

Additionally, a simplification was introduced based on the assumption that the ribs are straight:

$$\frac{d\delta}{dx} = 0.$$

As a result, the equation was rewritten in the following form:

$$\frac{d}{dx}\left(\lambda\,\delta\,\frac{d\theta}{dx}\right) = \alpha\,\theta. \tag{5}$$

The associated boundary conditions can be expressed as:

$$x = 0 \rightarrow \theta = \theta_0;$$

$$x = x_{max} \rightarrow \frac{\partial\theta}{\partial x} = 0.$$

Next, dimensional analysis was applied to the problem, with the temperature θ expressed as a function of the parameters in the original equation:

$$\theta = f(\lambda, \delta, \alpha, x, \theta_0).$$

In this case, the total number of variables is $n = 6$. Among them, it is possible to select k = 3 variables with independent dimensions: mass (m), temperature (K) and power (W). According to the Buckingham π-theorem (Ting, 2022), the relationship can be reduced to $n - k = 3$ dimensionless parameters. By selecting θ_0, λ *and* δ as the repeating (base) variables, the following dimensionless groups were obtained:

$$\Pi_0 = \frac{\theta}{\theta_0} = R;\ \Pi_1 = \frac{x}{\delta} = z;\ \Pi_2 = \frac{\alpha\,\delta}{\lambda} = Bi;$$

$$\Pi_0 = \varphi(\Pi_1, \Pi_2).$$

In the resulting equation, Bi represents the Biot number – a well-known similarity criterion defined as the ratio of the wall's thermal resistance, δ/λ, to the thermal resistance associated with heat transfer from the surface, $1/\alpha$.

To establish a relationship between Π_0 and the dimensionless groups Π_1 and Π_2, the variables from equation (5) were re-expressed in terms of these new dimensionless parameters:

$$\frac{x}{\delta} = z \rightarrow \partial z = \frac{\partial x}{\delta} \rightarrow \partial x = \delta\,\partial z \rightarrow \frac{\partial}{\partial x} = \frac{1}{\delta}\frac{\partial}{\partial z},$$

$$\frac{\theta}{\theta_0} = R \rightarrow \partial R = \frac{\partial\theta}{\theta_0} \rightarrow \partial\theta = \theta_0\,\partial R.$$

After substitution and simplification, the equation was obtained in the following form:

$$\frac{1}{Bi}\frac{\partial^2 R}{\partial z^2} = R. \tag{6}$$

The corresponding boundary conditions were written as:

$$z = 0 \rightarrow R(0) = 1;$$

$$z = z_{max} \rightarrow \frac{\partial R}{\partial z} = 0.$$

Next, numerical modeling of heat conduction within the fin was carried out. As shown in Figure 3, preliminary calculations confirmed the relevance of the problem, as a noticeable difference was observed between the surface temperature of the fin and the average temperature across its cross section.

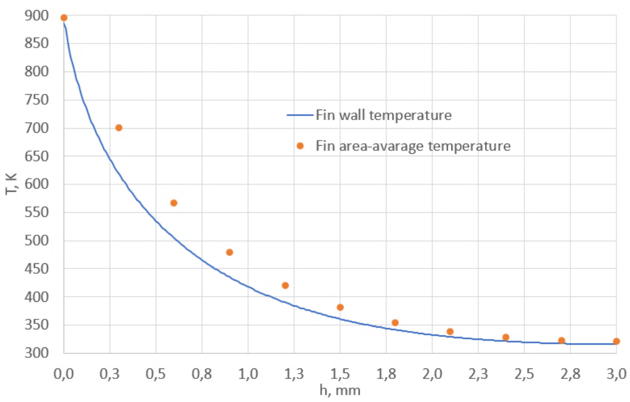

Figure 3 – Temperature variations along the wall (Source: Authors)

To assess the influence of various cooling channel parameters on the observed discrepancy, a series of parametric calculations was performed. The parameters varied included fin thickness, the thermal conductivity of the fin material, and the heat transfer coefficient. Table 1 presents the extracted heat fluxes obtained from Ansys Fluent (q_a) and from the derived equation (q_e), along with the corresponding error (Δ). The calculations were performed for the configurations shown in Figbre 2.

Table 1. Summary of calculation results (Source: Authors)

№	δ, mm	λ, W/(m·K)	α, W/(m²·K)	q_a, kW	q_e, kW	Δ, %
1	0.5	20	5	5.9	6.0	1.0
2			15	10.1	10.4	2.5
3			30	14.0	14.7	4.6
4		300	5	15.1	15.1	0.1
5			15	35.0	35.1	0.3
6			30	54.2	54.4	0.5
7	1.25	20	5	8.8	9.1	2.5
8			15	15.5	16.4	5.6
9			30	21.1	23.2	9.3
10		300	5	16.6	16.7	0.3
11			15	43.6	43.9	0.8
12			30	74.0	75.0	1.3

As shown in the table, the error exceeds 5% in some calculation cases, which is considered unsatisfactory. Figure 4 illustrates the influence of the fin material's thermal conductivity and the fin width on the non-uniformity of the temperature distribution across the cross section. The analysis indicates that the error associated with this effect increases with greater fin thickness and higher heat transfer coefficients, as well as with lower thermal conductivity of the fin material. These parameters collectively define the Biot number, which appears in the derived heat conduction equation for the fin.

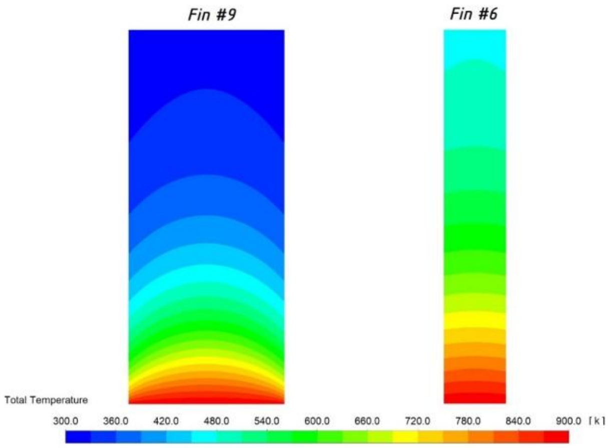

Figure 4 – Results of numerical simulation (Source: Authors)

To derive a relationship that accounts for the influence of temperature non-uniformity across the fin cross-section on the finning coefficient, the heat conduction equation was sought in the following form:

$$\frac{1}{Bi^*}\frac{\partial^2 R}{\partial z^2} = R,$$

where Bi^* denotes an effective Biot number. Based on the results of the numerical experiment, this effective Biot number was defined as:

$$Bi^* = Bi\,(q_e/q_a)^2,$$

where q_a is the heat flux obtained from the Ansys Fluent simulation and q_e is the heat fluxes calculated by integrating equation (6):

$$q_e = \alpha\,\delta_r\,\theta_0 \int_0^{z_{\max}} R(z)\,dz. \qquad (7)$$

Since the effective Biot number, Bi^*, depends on the calculated Biot number, its relationship was assumed to take the following form:

$$Bi^* = C\,Bi^m.$$

Since the target function has the form of equation (1), the previously described regression analysis method was used to determine the unknown coefficients. The resulting values of the coefficients were: $C = 0.853$, $m = 1.04$.

The resulting correlation was extended using equation (6). As a result, based on the numerical simulations conducted in Ansys Fluent, the following relationship was obtained:

$$0.853\,Bi^{-1.04}\frac{\partial^2 R}{\partial z^2} = R, \qquad (8)$$

Next, we derive an expression for calculating the finning coefficient, incorporating the previously obtained relationship. By definition, the finning coefficient is the ratio of the heat flux through a finned surface to that through a smooth wall:

$$\eta_r = \frac{q_r}{q_s}, \qquad (9)$$

The heat flux through smooth walls is calculated using the Richman equation:

$$q_s = \alpha\,R(0)\theta_0\,s = \alpha\,\theta_0\,s; \qquad (10)$$

To determine the heat flux through the fin in equation (7), the standard function R from equation (6) was replaced with a new function (8) that accounts for temperature non-uniformity:

$$q_\delta = \alpha\,\delta_r\,\theta_0 \int_0^{z_{max}} \left(0.853\,Bi^{-1.04}\frac{\partial^2 R}{\partial z^2}\right) dz. \qquad (11)$$

By substituting expressions (10) – (11) into equation (9), and considering that the rib covers only part of the wall surface, we obtain a formula for calculating the finning coefficient that accounts for transverse temperature non-uniformity in the rib cross section. After some simplifications, the final expression for the finning coefficient is given as follows:

$$\eta_r = \frac{\delta_r \int_0^{z_{mzx}} \left(0.853\,Bi^{-1.04}\frac{\partial^2 R}{\partial z^2}\right) dz + (s - \delta_r)}{s}.$$

Discussion

The obtained correlation was subsequently verified. In this study, the results of numerical modeling in Ansys were used as the reference, which is a reasonable assumption given that modern CAE systems can accurately simulate a wide range of physical processes. One limitation of the study is the small sample size used for the comparison, which will be addressed in future work. A visualization comparing the deviations between the classical method, the new equation, and the numerical modeling results is shown in Fig. 5.

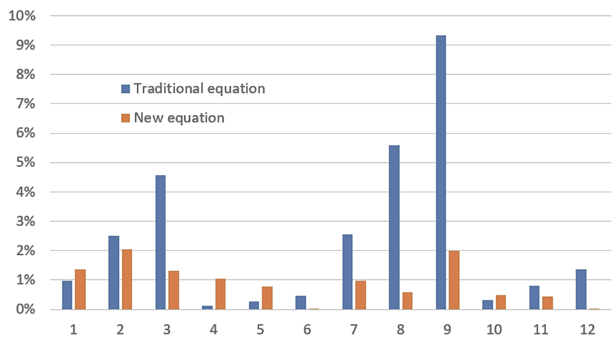

Figure 5 – Variation of the calculated error when applying the derived equation (Source: Authors)

For the cases considered (see Figure 2, Table 1), this equation yields an error of no more than 2%, representing a reduction in error by more than a factor of 4.5. Based on this comparison, obtained correlation can be recommended for use in practical calculations of cooling liquid propellant rocket engine chambers.

Conclusions

This study presents a new approach to determining the finning coefficient that accounts for the non-uniform temperature distribution across the fin cross-section. To achieve this goal, the previously derived heat conduction equation for a straight fin was first transformed into a dimensionless form using similarity theory. This transformation yielded key dimensionless parameters, including the dimensionless temperature, dimensionless coordinate, and the Biot number.

Subsequently, a series of numerical experiments were conducted to investigate heat conduction in the fins, using the Ansys Fluent CAE system for modeling. Based on the analysis of the simulation results, the error of the equation – caused by the transverse temperature non-uniformity in the fin cross-section – was estimated. For the cases considered, this error reached up to 9.3%.

Next, using the dimensionless criteria, a modified heat conduction equation was derived for the fin, incorporating the effect of transverse temperature non-uniformity. The coefficients of this equation were determined through regression analysis based on previously obtained numerical simulation results. Subsequent validation calculations demonstrated that, for certain fin configurations, the use of the proposed equation reduces the computational error by more than a factor of 4.5. The final equation was integrated into the expression for calculating the finning coefficient.

As a result, this study presents an original mathematical model that accounts for temperature non-uniformity across the fin cross-section. This improvement enhances calculation accuracy and can reduce the time required for the design and testing of components. Considering the above, the proposed approach is recommended for use in the thermal analysis of cooling channels in the chambers and gas generators of liquid rocket engines.

A key limitation of the proposed model is its assumption of fins with constant thickness. This restricts its applicability to advanced 3D-printed fin geometries with complex shapes. Future research will aim to refine the model to more accurately capture temperature non-uniformity within the fin cross-section. In particular, the influence of variable fin geometry will be explored to develop a more general and broadly applicable model.

References

Atefi, E., & Naraghi, M. H. (2019). Optimization of regeneratively cooled rocket engines cooling channel dimensions. In *AIAA Propulsion and Energy 2019 Forum* (p. 3938). https://doi.org/10.2514/6.2019-3938

Bilohurov, S., Myshenko, O., & Bardakov, S. (2024). Experimental Study of Throttling of Solid Fuel Direct-Flow Steam Boilers [In Ukrainian]. *Challenges and Issues of Modern Science, 2*, 72-77. https://cims.fti.dp.ua/j/article/view/129

Bucharskyi, V., Zhang, L.-H., & Wan, Y.-L. (2018). Improvement in Time Efficiency in Numerical Simulation for Solid Propellant Rocket Motors (SPRM). *Journal of Propulsion Technology, 39* (1), 92-99. http://dx.doi.org/10.13675/j.cnki.tjjs.2018.01.010

Fagherazzi, M., Santi, M., Barato, F., & Pizzarelli, M. (2023). A Simplified Thermal Analysis Model for Regeneratively Cooled Rocket Engine Thrust Chambers and Its Calibration with Experimental Data. *Aerospace, 10*(5), 403. https://doi.org/10.3390/aerospace10050403

Forsberg, C. H. (2021). Steady-state conduction. In *Heat Transfer Principles and Applications* (pp. 57–120). Academic Press. https://doi.org/10.1016/b978-0-12-802296-2.00003-2

Jeong, W., Jang, S., & Kim, H.-J. (2023). Characteristics of a Heat Exchanger in a Liquid Rocket Engine Using Conjugate Heat Transfer Coupling with Open-Source Tools. *Aerospace, 10*(12), 983. https://doi.org/10.3390/aerospace10120983

Kim, S.-K., Joh, M., Choi, H. S., & Park, T. S. (2014). Effective Modeling of Conjugate Heat Transfer and Hydraulics for the Regenerative Cooling Design of Kerosene Rocket Engines. *Numerical Heat Transfer, Part A: Applications, 66*(8), 863–883. https://doi.org/10.1080/10407782.2014.892396

Kose, Y. M., & Celik, M. (2023). Regenerative Cooling Comparison of LOX/LCH4 and LOX/LC3H8 Rocket Engines Using the One-Dimensional Regenerative Cooling Modelling Tool ODREC. *Applied Sciences, 14*(1), 71. https://doi.org/10.3390/app14010071

Leonardi, M., Pizzarelli, M., & Nasuti, F. (2019). Analysis of thermal stratification impact on the design of cooling channels for liquid rocket engines. *International Journal of Heat and Mass Transfer, 135*, 811–821. https://doi.org/10.1016/j.ijheatmasstransfer.2019.02.028

Lv, J., Du, G., Jin, P., & Li, R. (2023). Heat Transfer Analysis and Structural Optimization for Spiral Channel Regenerative Cooling Thrust Chamber. *International Journal of Aerospace Engineering, 2023*, 1–17. https://doi.org/10.1155/2023/8628107

Mitikov, Y., & Sedchenko, M. (2023). Critical analysis of helium gas bottle inflation systems for rocket engine fuel tanks [In Ukrainian]. *Challenges and Issues of Modern Science, 1*, 117-125. https://cims.fti.dp.ua/j/article/view/23

Mohr, D. L., Wilson, W. J., & Freund, R. J. (2022). Linear Regression. In Statistical *Methods* (pp. 301–349). Academic Press. https://doi.org/10.1016/b978-0-12-823043-5.00007-2

PTC Inc. (2025). *Mathcad: Math Software for Engineering Calculations*. PTC Inc. https://mathcad.com

Sichler, E., Montes, J. D., & Chandler, F. O. (2018). One dimensional thermal steady state analysis and procedure for a low-pressure liquid oxygen and liquid methane rocket engine. In *2018 Joint Propulsion Conference* (p. 4602). https://doi.org/10.2514/6.2018-4602

Sliusariev, V., & Bucharskyi, V. (2024). Development of a mathematical model for the cooling channel of a liquid propellant rocket engine's chamber with respect for variations in coolant density. *Eastern-European Journal of Enterprise Technologies, 6* (1 (132)), 14–20. https://doi.org/10.15587/1729-4061.2024.316236

Sliusariev, V., & Bucharskyi, V. (2024). Mathematical model for heat transfer in variable thickness fins for rocket engines. *Challenges and Issues of Modern Science, 3*, 48–54. https://cims.fti.dp.ua/j/article/view/234

Sukachevskyi, V., & Shevtsov, V. (2024). Application of computer-integrated technologies in rocket engine design [In Ukrainian]. *Challenges and Issues of Modern Science, 3*, 62–73. https://cims.fti.dp.ua/j/article/view/201

Ting, D. (2022). Dimensional analysis. In *Thermofluids* (pp. 181–198). Academic Press. https://doi.org/10.1016/b978-0-323-90626-5.00003-3

Tokarskyi, I., & Habrinets, V. (2024). Design Analysis and Calculation Methods for Furnace Oil Burners in Heating Systems [In Ukrainian]. *Challenges and Issues of Modern Science, 3*, 55–61. https://cims.fti.dp.ua/j/article/view/225

Vekilov S. S., Lipovskyi V. I., Marchan R. A., & Bondarenko O. E. (2021). Distinctive features of SLM technology application for manufacturing of LPRE components. *Journal of Rocket-Space Technology, 29*(4), 112-123. https://doi.org/10.15421/452112

Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D., Burovski, E., Peterson, P., Weckesser, W., Bright, J., van der Walt, S. J., Brett, M., Wilson, J., Millman, K. J., Mayorov, N., Nelson, A. R. J., Jones, E., Kern, R., Larson, E., … Vázquez-Baeza, Y. (2020). SciPy 1.0: fundamental algorithms for scientific computing in Python. *Nature Methods, 17*(3), 261–272. https://doi.org/10.1038/s41592-019-0686-2

Williams, M. (2023). *SpaceX sets new record with successful test-firing of 31 Raptor engines*. Universe Today. https://www.universetoday.com/articles/spacex-sets-new-record-with-successful-test-firing-of-31-raptor-engines

Wolfram Research, Inc. (2025). *Mathematica 12.3*. Wolfram Research, Inc. https://www.wolfram.com/mathematica

Xu, B., Chen, B., Peng, J., Zhou, W., & Xu, X. (2023). A Coupled Heat Transfer Calculation Strategy for Composite Cooling Liquid Rocket Engine. *Aerospace, 10*(5), 473. https://doi.org/10.3390/aerospace10050473